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Coherent coupling of a trapped electron to a distant superconducting microwave cavity

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We investigate theoretically the coupling of a single electron in a planar Penning trap with a remote superconducting microwave cavity. Coupling frequencies around $\Omega = 2\pi \cdot 1$ MHz can be reached with resonators with a loaded quality factor of $Q = 10^5$, allowing for the strong coupling regime. The electron and the cavity form a system of two coupled quantum harmonic oscillators. This is a hybrid and linear microwave quantum network. We show that the coherent interaction can be sustained over distances of a few mm up to several cm. Similar to classical linear MW circuits, the coherent quantum exchange of photons is ruled by the impedances of the electron and cavity. As one concrete application, we discuss the entanglement of the cyclotron motions of two electrons located in two separate traps.

Cryogenic Penning traps allow for a very accurate control of the dynamics of a trapped electron, at the fundamental level of quantum jumps between the Fock states of the harmonic trapping potential¹. The particles can be captured for very long periods (months); the continuous Stern-Gerlach effect permits the detection and manipulation of the electron's spin² and the Purcell effect enhances the coherence time of its motional quantum state³. The latter can be monitored non destructively, thereby effectively performing a quantum non demolition (QND) measurement of microwave quanta¹. Hence, cryogenic Penning traps are excellent quantum laboratories and trapped electrons are solid candidates for quantum technology⁴⁻⁶. We are developing a trapped electron for quantum metrology applications, specifically as a transducer of quantum microwave (MW) radiation⁷. For this we use the “geonium chip” planar Penning trap⁸⁻¹⁰. The electron can operate both, as a detector and also as an emitter of MW photons. Being a quantum harmonic oscillator, in principle its state can be mapped one to one onto the quantum state of a single mode MW radiation field¹¹. This makes the electron a linear and reversible quantum microwave transducer, unlike single MW photon counters based upon three-level systems^{12,13} and similar technologies. In those cases only MW fields consisting of one (and only one) photon can be observed, and these are irreversibly lost after the detection. In contrast, the trapped electron might “witness” a more complex quantum microwave field and further reuse or redistribute it to other devices within a MW quantum network. In this letter we theoretically investigate the basic scheme for such reversible and QND measurements of MW radiation. The quantum object to be “measured” is assumed to be a distant superconducting microwave cavity, coupled to the electron through a transmission-line of some finite length.

Fig. 1 (a) shows a geonium chip with its five basic trapping electrodes. These result from the projection of a 5 pole cylindric Penning trap onto a flat surface⁸. The buried wires

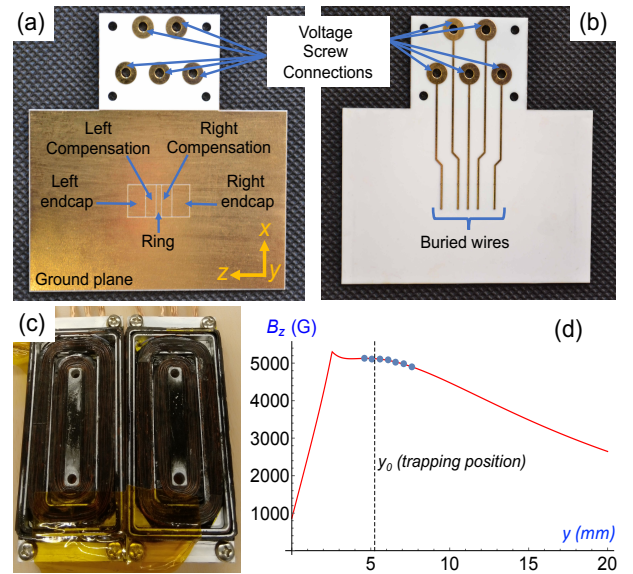


FIG. 1. (a) Top view of “geonium chip” with the trap’s electrodes. (b) Chip’s rear side showing the buried wires. (c) Magnetic field source made of NbTi wire. (d) Example of measured magnetic field (dots) and fitted Biot-Savart functions (continuous curve).

shown in fig. 1 (b) provide the required DC trapping voltages, and also coupling for RF signals. They are connected through vias to the trap’s electrodes. A static magnetic field, $\vec{B} = B_0 \hat{u}_z$, forces the electron to follow a closed (cyclotron) orbit around its axis. The magnetic field source is shown in Fig. 1 (c). It is made of NbTi superconducting wire and spans about $\sim 10 \times 10$ cm². Its construction and calibration have been described in^{14,15}. It is placed underneath the geonium chip and is constantly powered with DC current supplies. An example of a measured field is shown in Fig. 1 (d). Homogeneous magnetic fields up to 0.5 T (= 5000 G) at the electron’s trapping position can be reached with this source. A new magnetic source is under development¹⁵ which operates in persistent mode and is magnetised with a specially devised flux pumping technique¹⁶. The electrons can be captured at some

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height y_0 above the central conducting strip. The value of y_0 is determined by the applied DC voltages⁸. The particle's motion consists of three independent oscillations: the mentioned cyclotron motion, with frequency ω_p , and the axial and magnetron motions. The latter two will not be further considered in this letter. At $B_0 = 0.5$ T, we have $\omega_p/2\pi = 13.99$ GHz⁸.

Superconducting microwave resonators can be fabricated in different shapes, for instance as 3D cavities made of aluminium¹⁷ or copper with a tin coating¹⁸, reaching internal quality factors as high as $Q \sim 10^9$, and also as flat, chip devices, such as coplanar-waveguide (CPW) cavities, achieving quality factors $Q \in [10^5, 10^6]$ ^{19,20}. While our electron might be coupled to either kind of resonators, for simplicity we concentrate on the CPW cavities; our analysis will apply also for other types of MW resonators. Their high Q allows for transferring quantum information between different components in a microwave quantum circuit, such as in circuit-quantum electrodynamics (cQED)²¹. Due to the long coherence time of their spin state, electrons in different forms, such as trapped on the surface of liquid Helium, within molecular ions or as spin ensembles in solid state systems, have been proposed and are being tested as quantum memories for cQED^{22–26}. A review²⁷ of other atomic systems proposed as potential quantum memories for cQED has been published. A scheme for interfacing electrons in a Paul trap with superconducting qubits has also been discussed²⁸. Our trapped electron can be coupled to a microwave quantum circuit via the interaction of the photons with its cyclotron motion or its spin. The latter is very weakly coupled, hence, we focus on the former.

The electrodes of the geonium chip define the central conducting strip of a CPW transmission-line⁸, where MW photons can exist in either one of the two possible propagation modes²⁹, the odd or the even mode, of a CPW line. Thus, our chip might be designed and operated also as a planar microwave cavity. However, the quality factor of a superconducting CPW resonator is reduced by three to four orders of magnitude in the presence of a magnetic field¹⁹. This makes it inadequate for storing MW photons. Furthermore, we want to investigate the QND measurement of the quantum state of a microwave system at an arbitrary distant location. Thus, we assume our chip is coupled to a *remote* CPW superconducting cavity, as sketched in Fig. 2 (a). The trap and the cavity are connected through a superconducting transmission-line of some finite length L_{TL} . The geonium chip confines the magnetic field into the trapping region, which decays rapidly away from it, as shown in Fig. 1 (d). Hence, \vec{B} does not reduce the quality factor of the superconducting cavity. The interaction between the distant MW quantum device (the cavity) and the electron is mediated by photons. For simplicity, we assume the photons when arrived at the geonium chip exist in one mode of the CPW shaped by the trap's electrodes, as mentioned above. However, in general, this radiation can be delivered to the trapped electron by other types of transmission-lines coupled to the electrodes; or to a small antenna fabricated in the chip, such as the antennas used in near field scanning microwave microscopes³⁰.

The interaction of a trapped electron with a radio frequency (RF) resonator made of lumped elements was first investigated

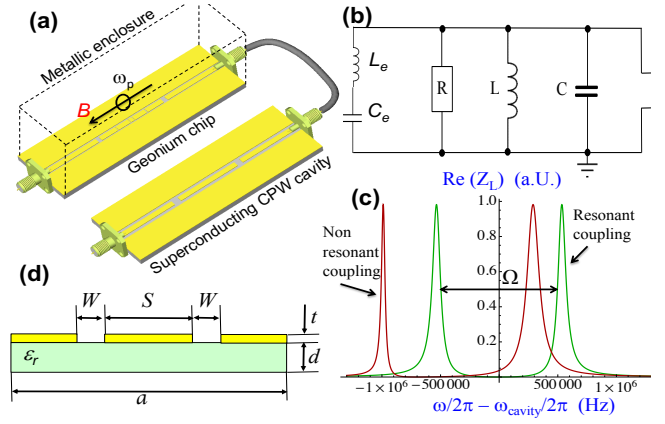


FIG. 2. (a) Sketch of the geonium chip connected to a remote superconducting CPW cavity. The open ends avoid photon leaks by reflecting the microwaves inside the system. (b) Equivalent electric circuit of the coupled electron and CPW cavity. (c) Impedance of the cavity + electron's cyclotron. This example assumes strong, coherent coupling. (d) Transverse dimensions of a CPW transmission-line.

by H. Dehmelt³¹. This interaction is commonly employed for the electronic detection of the trapped particles and can be described as the coupling of two equivalent electric circuits³¹. In our system of Fig. 2 (a), the RF resonator is substituted by the superconducting CPW microwave cavity. The latter is equivalent to a parallel tank circuit, with inductance L , capacitance C and losses modelled by the resistance R . Moreover, the electron's cyclotron motion behaves as a series tank circuit, with equivalent inductance L_e and capacitance C_e ³². The electron-cavity interaction corresponds to the coupling in parallel of their equivalent electric circuits³¹, as represented in Fig. 2 (b). Hence, the system's overall impedance is $\frac{1}{Z_L(\omega)} = \frac{1}{Z_{\text{cavity}}(\omega)} + \frac{1}{Z_e(\omega)}$, where $Z_{\text{cavity}}(\omega)$ is the cavity's input impedance seen from the trap and $Z_e(\omega) = i\omega L_e + \frac{1}{i\omega C_e}$ the electron's cyclotron impedance. $Z_L(\omega)$ delivers a resonance spectrum as plotted in Fig. 2 (c). In the case of resonant coupling, two symmetric peaks appear around the cavity's resonance frequency, ω_{cavity} . The energy exchange rate between the particle and resonator is given by the width Ω of the dip between both maxima. In Penning trap experiments such as^{33–35}, typical values of the used RF resonators³⁶ are $Q \leq 10^4$. Thus, in those cases, once delivered to the resonator, the particle's energy is irreversibly dissipated almost instantly³⁷ and $\tau = 1/\Omega$ is denoted as the “resistive cooling time constant.”

In order to get the geonium chip down to 80 mK we use a miniature adiabatic demagnetisation refrigerator³⁸. This will also require MW attenuators^{39,40} to avoid residual thermal photons entering into the system. With these, at 80 mK the number of blackbody photons at 13.99 GHz is sufficiently low for both constituents of Fig. 2 (a) to reach the ground state¹. This system becomes a quantum circuit and can be excited with a quantum of current, i.e. one single microwave photon. The photon then oscillates between both components at the coupling frequency Ω . From the general expression of

τ^{32} , we have $\Omega(\delta) = \frac{|Z_{\text{cavity}}(\delta)|}{L_e}$, with $\delta = \omega_{\text{cavity}} - \omega_p$. The coupling Ω is maximum when the electron and cavity are resonant $\omega_{\text{cavity}} = \omega_p$. In that case the impedance of the cavity is $Z_{\text{cavity}}(0) = R = Q \omega_{\text{cavity}} L^{41}$. When $\delta \neq 0$, $|Z_{\text{cavity}}(\delta)|$ decreases, the coupling strength is reduced and the two maxima become asymmetric, as shown in Fig. 2 (c). For the e^- -cavity coupling to be coherent, Ω must be faster than the photon loss rate of the resonator, responsible for the irreversible energy dissipation causing the “resistive cooling.” The cavity’s photon loss rate is given by the width of its resonance: $\Gamma = \omega_{\text{cavity}}/Q$. Hence, the condition for coherent coupling is: $\frac{\Omega}{\Gamma} \gg 1$. For the resonant case, $\delta = 0$, we have:

$$\frac{\Omega}{\Gamma} = Q^2 \frac{L}{L_e} \gg 1. \quad (1)$$

In Eq. (1), Q is the quality factor of the loaded MW cavity, as seen by the electron, that is: including possible photon losses along the connecting transmission-line. It must be observed that, as sketched in Fig. 2 (a), the trap is enclosed within a rectangular metallic box¹⁴. This also acts as a microwave resonator far detuned from ω_p ¹⁰. Hence, it prevents the electron from emitting cyclotron radiation into free space or into any other modes but those of the trap’s CPW line, coupling to the remote MW cavity.

The ratio Ω/Γ of Eq. (1) can be calculated with the general expression^{7,32} $L_e = \frac{m}{q^2} D_{\text{eff}}^2$. The symbols q and m represent the charge and mass of the electron, respectively. The “effective coupling distance”, D_{eff} , is inversely proportional to the strength of the electric field at the position of the electron, $E_{\text{CPW}}(y_0)$, of a 1 V microwave propagating along the CPW. In general both quantities are related through³²: $E_{\text{CPW}}(y_0) = \frac{1V}{D_{\text{eff}}}$. For the slow axial motion ω_z , the corresponding RF electric field has been calculated using electrostatic techniques³². That approach must be modified for the electron’s cyclotron motion, which belongs to the MW domain. The electric field components $E_{\text{CPW}}^{\text{odd}}$ and $E_{\text{CPW}}^{\text{even}}$ have been computed analytically using special MW techniques²⁹. With those results, we obtain the cyclotron “effective coupling distances,” for each of the two possible CPW modes:

$$(D_{\text{eff}}^{\text{odd}})^{-1} = \frac{4}{\pi W} \sum_{n=1}^{\infty} \left\{ \frac{\sin(n\pi \frac{W}{a}) \sin(n\pi \frac{W+S}{a})}{\sqrt{n^2 + \frac{a^2}{\lambda_p^2} - \frac{a^2 v_p^2}{c^2}}} \right. \quad (2)$$

$$\left. \times \exp \left(-\frac{2\pi y_0}{a} \sqrt{n^2 + \frac{a^2}{\lambda_p^2} - \frac{a^2 v_p^2}{c^2}} \right) \right\}.$$

$$(D_{\text{eff}}^{\text{even}})^{-1} = \frac{4}{\pi W} \sum_{n=0}^{\infty} \left\{ \frac{\sin(\frac{2n+1}{2} \pi \frac{W}{a}) \cos(\frac{2n+1}{2} \pi \frac{W+S}{a})}{\frac{2n+1}{2}} \right. \quad (3)$$

$$\left. \times \exp \left(-\frac{2\pi y_0}{a} \sqrt{\left(\frac{2n+1}{2}\right)^2 + \frac{a^2}{\lambda_p^2} - \frac{a^2 v_p^2}{c^2}} \right) \right\}.$$

In Eq. (2,3) c is the speed of light in vacuum. The transverse dimensions of the CPW are defined in Fig. 2 (d). S is the width of the central conducting strip, W is the gap between the latter and the “ground planes” and a is the total chip’s width.

In the geonium chip, the wavelength λ_p of the radiation of cyclotron frequency ($\nu_p = \omega_p/2\pi$), depends on the substrate’s electric permittivity ϵ_r , its thickness d , the thickness of the conducting layer t and the dimensions S, W ⁴².

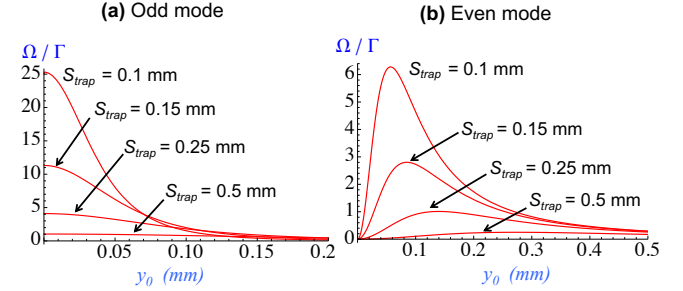


FIG. 3. Coupling frequency of one electron versus photon losses as a function of the trapping position; for various values of the CPW width in the trap chip. We assume $\delta = 0$ and $Q = 10^5$. Calculated with Eq. (2-3), truncating the series at $n = 60000$. (a) Odd Mode. (b) Even Mode.

In order to illustrate our system we have calculated in detail one example. It is shown in Fig. 3. We have assumed a distant superconducting $\lambda/4$ short circuited CPW microwave cavity⁴¹, with dimensions: $S_{\text{cavity}} = 10 \mu\text{m}$, $W_{\text{cavity}} = 4.8 \mu\text{m}$, $t_{\text{cavity}} = 200 \text{ nm}$, and a sapphire substrate of $d_{\text{cavity}} = 0.3 \text{ mm}$. These values are motivated by some real resonators^{19,20}. Our cavity has the characteristic impedance $Z_0 = 50 \Omega$, as obtained from previously compiled CPW design formulas⁴². Further, we assume a loaded quality factor (as seen by the electron) of $Q = 100000$. This corresponds to $\Gamma/2\pi = 140 \text{ kHz}$, resonance resistance $R = 6.37 \text{ M}\Omega$ and inductance $L = 0.72 \text{ nH}$. Moreover, we have assumed a geonium chip of the same conducting layer and with an alumina substrate of thickness 0.7 mm , as in Fig. 1 (a). With these, we compute Ω/Γ for 4 values of the width: $S_{\text{trap}} = 0.5, 0.25, 0.15$ and 0.1 mm , with gaps $W_{\text{trap}} = 59, 34, 22$ and $15 \mu\text{m}$, respectively. Each W_{trap} is adjusted to ensure that the CPW formed in the geonium chip has a characteristic impedance of $Z_0 = 50 \Omega$.

The increasing values of Ω/Γ for smaller S_{trap} observed in Fig. 3 (b,c) reflect the bigger strength, close to the chip’s surface, of the microwave electric field E_{CPW} when the “mode volume” of the CPW line in the trap is reduced. At very low positions, Ω/Γ is higher for the odd than for the even mode; the latter actually disappears at $y_0 \rightarrow 0$. This is due to the vanishing electric field component parallel to the conducting surface: $E_{\text{CPW}}^x \rightarrow 0$ at $y_0 \rightarrow 0$. The active mode, odd or even, depends on how the system is connected and grounded⁴². The electron can be positioned at $y_0 \geq 50 \mu\text{m}$, by carefully choosing the trapping voltages^{8,10}. At such low positions the electrostatic trapping potential becomes anharmonic⁸ and at 80 mK the expected relative frequency shifts can be estimated to $\frac{\Delta\omega_p}{\omega_p} \simeq 6 \times 10^{-11}$. Such fluctuations of ω_p are negligibly small. Therefore, when placed at $y_0 \geq 50 \mu\text{m}$, the electron’s cyclotron motion still behaves as a quantum harmonic oscillator. As shown in Fig. 3, at such low y_0 , the cavity and the cyclotron oscillator interact coherently. After exchanging quantum information, y_0 can be increased (typically to $0.5 - 1$

mm) where the trapping potential becomes harmonic. There ω_p can be measured accurately^{8,10} and the cyclotron quantum state read out and further manipulated.

The electron and the cavity are two coupled quantum harmonic oscillators. This is a linear system, where the coupling strength Ω is invariant under the number of photons. The coherent dynamic of two coupled quantum harmonic oscillators has been calculated¹¹. An equivalent system has been implemented with two ions held in two Paul traps at a distance of $40 \mu\text{m}$ ⁴³. In our case, the maximum cavity-electron separation permitting coherent coupling is bounded by the achievable loaded quality factor Q . The dependence $Q = Q(L_{\text{TL}})$ can be obtained from the transmission-line impedance equation⁴¹. Thus, at the trap, the cavity's unloaded input impedance $Z_{\text{cavity}}^0(\omega)$ is transformed into:

$$Z_{\text{cavity}}(\omega|L_{\text{TL}}) = Z_0 \frac{Z_{\text{cavity}}^0(\omega) \coth(\gamma L_{\text{TL}}) + Z_0}{Z_0 \coth(\gamma L_{\text{TL}}) + Z_{\text{cavity}}^0(\omega)}. \quad (4)$$

The imaginary part of the propagation constant, $\gamma = \alpha + i\beta$, shifts the cavity's resonance frequency. We therefore assume that the cavity, as seen from the trap, has been designed to be resonant with ω_p . The real part α accounts for the photon losses. The dissipation caused by the walls of the connecting transmission-line is linked to the superconductor surface resistivity: $R_S \leq \frac{5}{4\pi^2} \cdot 10^{-28} \omega^2 \text{ Ohm}$ (with ω given in rad/s)⁴⁴. At 13.99 GHz that is negligible when compared to the dielectric losses. The latter, in the case of a superconducting coaxial line operating at ω_p , give rise to the attenuation constant $\alpha = \omega_p \sqrt{\mu_0 \cdot \epsilon_{\text{TL}}} \tan \delta$ ⁴¹, with μ_0 = magnetic permeability, ϵ_{TL} = electric permittivity and $\tan \delta$ = loss tangent. With that α and with the same cavity as in Fig. 3, assuming²⁰ $Q_{\text{unloaded}} = 10^6$, we compute $Q = Q(L_{\text{TL}})$ from the spectra $Z_{\text{cavity}}(\omega|L_{\text{TL}})$. The results are in Fig. 4 (a).

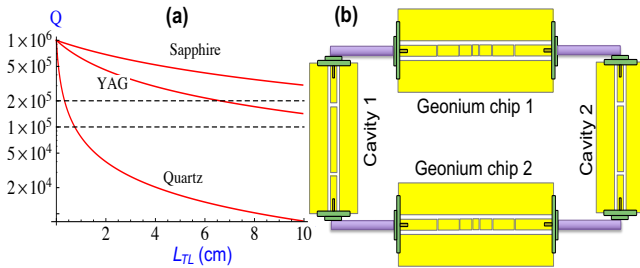


FIG. 4. (a) Loaded Q versus trap-cavity distance. Calculated for 3 different dielectric materials in the coax line. (b) Two geonium chips connected to two different distant identical cavities.

We have assumed quartz with $\epsilon_{\text{TL}} = 4.64$ (relative), $\tan \delta = 3 \cdot 10^{-6}$; YAG with $\epsilon_{\text{TL}} = 10.40$, $\tan \delta = 1 \cdot 10^{-7}$ and sapphire with $\epsilon_{\text{TL}} = 9.4$, $\tan \delta = 4 \cdot 10^{-8}$, all measured values at ~ 10 K⁴⁵. From the figure we observe that, for quartz, the loaded quality factor drops below 10^5 at a distance of $L_{\text{TL}} \simeq 0.8$ cm, while both YAG and sapphire sustain $Q > 10^5$ at separations above 10 cm. Within that range of distances the coherent e^- -cavity coupling might be achieved, as shown in Fig. 3. It must be observed that in this discussion we have assumed one or small numbers of MW photons. When the quantum

state of the exchanged MW signal consists of a large number of photons, then the coherence or the “quantum nature” of the state will in general degrade more rapidly than for low photon numbers⁴⁶. Bigger number of photons will make the exchanged MW signal increasingly non monochromatic and, in that case, the dispersion of the coupling transmission-line will need to be considered.

Through the coherent interaction of the trapped electron with a remote superconducting resonator, the former can be further coupled to other components in a microwave quantum network. The electron's cyclotron oscillator enables the linear mapping of the quantum state of any multi photon microwave field onto an atomic degree of freedom. This is possible while keeping the photons stored within the cyclotron quantum state for periods of the order of a minute¹. The linearity of the electron also makes the computation of the coupling with any other quantum devices in a MW network straightforward: the coupling strength Ω is simply given by the transformed impedance (through eq. 4) of that device as seen by the electron. Furthermore, in contrast to systems where the atomic species are attached to the CPW microwave cavity^{22–26}, our electron quantum MW transducer and the cavity are two fully independent devices. This makes increasingly complex network topologies possible, enabling many applications beyond quantum MW memories. A fundamental one is the entanglement of two electrons stored in different traps. This can be achieved with a configuration as sketched in Fig. 4 (b). There, two traps are connected to two cavities of same resonance frequencies and quality factors. Cyclotron-cyclotron entanglement occurs through a similar mechanism as demonstrated with two laser cooled ions in distant Paul traps⁴⁷. For that, the cyclotron oscillators must be prepared in the first excited state, $n_p = 1$, with both particles at a high y_0 , i.e. initially invisible to the cavities, due to the vanishing coupling (Fig. 3). Such state initialisation can be achieved by pumping photons from a very weak source connected to the traps, while monitoring the quantum number n_p non destructively¹. Thereafter the electrons are simultaneously dropped for a short period of time to a very low position, around $50 \mu\text{m}$, hence switching on briefly the strong coupling with the MW cavities. Hence, in our system, the unavailable fast spontaneous emission (essential in the Paul traps experiment⁴⁷) is substituted by the rapidly switchable strong coupling to the remote resonators. This forces both cyclotrons to deliver the microwave photons simultaneously (in principle with equal probability) to any of the cavities. The irreversible detection of the emitted photons, either in cavity 1 or 2 (for instance as in ref⁴⁸), erases the “which way” information, thereby entangling the particles. The entanglement of electrons stored in one trap has been proposed for quantum metrology applications, such as the enhancement of the g-factor measurement⁴⁹. The trapped electron quantum microwave transducer can also be used in other practical applications, for instance as an ultra sensitive MW detector for near field scanning microwave microscopes⁵⁰.

ACKNOWLEDGMENTS

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The data that supports the findings of this study are available within the article.

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